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ON THE SPATIAL DISTRIBUTION OF MUON FLUXES IN BROAD AIR SHOWERS

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SUMMARY

The pion generation function with respect to atmospheric depth in broad air showers is found on the basis of experimental data on spatial distribution of muon fluxes in broad air showers and on the distribution of pion transverse momenta in nuclear interaction events. Considered here is the simplest case of fluctuations of the spatial distribution function of muon fluxes in broad air showers (BAS).

* *

The currently available experimental data on the mean spatial distribution of muon fluxes with energy $E \geqslant 5$ hev and $E \geqslant 10$ hev ([1], [3]) in broad air showers (BAS) at sea level, and the experimental data on the distribution of transverse pion pulses in nuclear interaction events with energy $10^{11} \to 10^{12}$ ev [4] allow the hope to obtain information on the effective spectrum (responsible for the mean spatial distribution of muons) of generation in depth of charged pions in the composition of BAS. Knowing the effective spectrum of pion generation, we may bring up the question of possible fluctuations of the form of this spectrum, and by the same token of possible fluctuations of functions of spatial distribution of muons in BAS. This last question is important in the study of fluctuations of the total muon flux in BAS [5, 6].

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* O PROSTRANSTVENNOM RASPREDELENII POTOKOV MYUONOV SHIROKIKH ATMOSFERNYKH LIVNYAKH.

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1.- ANALYSIS OF THE MEAN SPATIAL DISTRIBUTION OF THE MUON FLUX IN BAS

The experimental data on the mean spatial distribution of muon fluxes with energies $\geqslant 5$ and $\geqslant 10$ hev in BAS with a total number of particles $N = 10^6$, obtained in the works [1 - 3] (sea level; the data of reference [2] are reduced to the value $N = 10^6$ according to the law $N_\mu \sim N^{0,85}$) *. We shall set up the following problem :

a) when computing the spatial distribution of muons, we may neglect the Coulomb scattering of pions and muons (see [1]);

b) all the pions are generated in the air-shower core;

c) the energy of the pion E, giving at decay a muon with energy E_μ, is always $E = 1.4\ E_\mu$.

Defining the spectrum of charged pion generation $f(E, H)\, dE dH$, H being the height above the observation level in meters) and the spectrum of transverse pulses of the generated pions $\varphi(p_\perp)\, dp_\perp$, and utilizing the equality $p_\perp = rp / H = rE / cH$ (r being the distance from the shower's core, we obtain the following expression for the flux density of muons with energy $E \geqslant E_\mu$ at the distance r from the axis of the shower :

$$\rho(r, E_\mu) = \int_0^\infty dH \int_{(E_\mu + E_i) 1.4}^\infty dE f(E, H) W_{\pi\to\mu}(E, H) \varphi\left(\frac{rE}{cH}\right) \frac{W_\mu(E, H) E}{2\pi r c H} =$$

$$= \int_0^\infty dH \int_{(E_\mu + E_i) 1.4}^\infty dE f(E, H) L_{вз} \left[1 - \exp\left(-\frac{H}{L_{эфф}}\right)\right] \times$$

$$\times \varphi\left(\frac{rE}{cH}\right) \exp\left(-\frac{H}{L_{p\mu}}\right) \frac{E}{2\pi r c H (L_{вз} + L_{p\pi})}. \qquad (1)$$

Here $L_{вз}$ is the path for pion interaction (computed starting from the path for the interaction of $\lambda = 80\ g/cm^2$ for an atmosphere density dependence on H, according to data of the work [10], $L_{p\pi} = Ec / m_\pi c^2$ is the path for the decay of charged pions, $L_{p\mu} = Ec / 1{,}4 m_\mu c^2$ is the path of the decay of muons, E_i are the ionization losses of the pion and muon from the place of generation to the observation level.

The expression (1) was utilized for the search of a form of the function $f(E, H)$, that would give a spatial distribution function of muons, agreeing well with the experimental values of muon density fluxes $\rho(r, E_\mu)$,

* Data from other works, plotted in Fig. 1 , are discussed below.

($E_\mu = 5$ and 10 hev), plotted in Fig. 1. The function $f(E, H)$ was sought for in the form

$$f(E,H) = bE^{-\gamma-1}F(H)$$

by proper assortment of various values of γ and of the function $F(H)$. The distribution of the transverse pulses of ions was taken in the form

$$\varphi(p_\perp) = p_\perp^2 \exp(-p_\perp / a) / 2a^3, \quad a = 0{,}1 \text{ hev / c}$$

in agreement with reference [4].*

The function $F(H)$ was sought for in the form of cascade curve $F(x(H))$, $x(H)$ being the depth of the atmosphere. with various positions of the maximum x_m and various values of the path of exponential absorption Λ behind the maximum. As a result of tests of various curves $F(x(H))$ and values of the exponent of the energy spectrum of pions, it was found, that the best agreement with the experimental data is given by the values $\gamma = 1{,}3 \pm 0{,}1$ and the function $F(x(H))$ with $\Lambda = 200 \div 250$ g/cm^2 and $x_m = 250$ g/cm^2. A family of curves $F(x)$ with absorption path $\Lambda = 250$ g/cm tests during the computation, are plotted in Fig. 2.

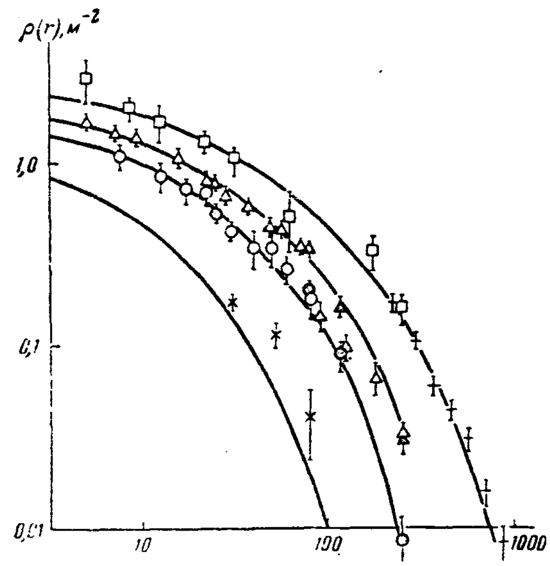

Fig. 1. - Spatial distribution of muon fluxes in BAS ($N = 10^6$) after the data of various authors:

□ — $E_\mu = 0{,}5$ Гэв [7], + — $E_\mu = 0{,}5$ Гэв [8], △ — $E_\mu = 4{,}5$ Гэв [2], ▲ — $E_\mu = 5$ Гэв [1], ○ — $E_\mu = 10$ Гэв [3], ● — $E_\mu = 10$ Гэв [1], × — 40 Гэв [9] [Гэв means Hev]

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* The experimental distribution of transverse pulses, brought out in [4], may be also approximated by the function $\varphi(p_\perp) = p_\perp \exp(-p_\perp / a) / a^2$, where $a = 0.15$ hev/c. We conducted the calculation for this case also. It was found, that for the tested values of the exponent $\gamma = 1 \div 1.5$ and various functions $F(H)$, no satisfactory agreement can be obtained between the computed and the experimental distribution $\rho(r, E_\mu)$.

It was also tackled, to what extent the distribution $\rho(r, E_\mu)$ is sensitive to the existence of large transverse pulses of pions ($p_\perp > 1$ hev/c) that cannot be described by the approximate formulas $\varphi(p_\perp)$. It was found that for E_μ, equal to 5 and 10 hev, at distances $r \leq 250$ m and for the same probabilities of $p_\perp > 1$ hev/c appearance, that would correspond to the experimental distribution by $p_\perp$ from [4], the spatial distribution of $\rho(r, E_\mu)$ is not sensitive to the existence of $p_\perp > 1$ hev / c.

Plotted in Fig. 3 are the spatial distributions of muons $\rho(r, E_\mu)$, corresponding to the family of curves of Fig. 2 and $\gamma = 1.3$. The curves $\rho(r, E_\mu)$, giving the best agreement with the experiment (number 2, Fig. 3), are also plotted in Fig. 1. It was ascertained from the computation, that for the agreement with experimental data it is important that the value $\gamma = 1.3$ materialize in the lower half of the atmosphere. We may admit that in the upper half of the atmosphere $\gamma = 1$.

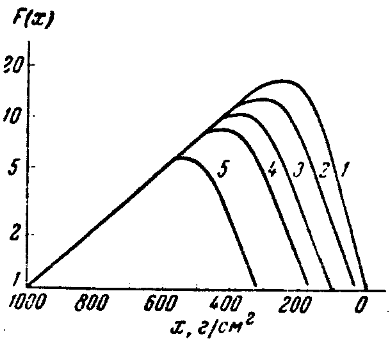

Fig. 2. - Functions F (x) utilized in the calculations

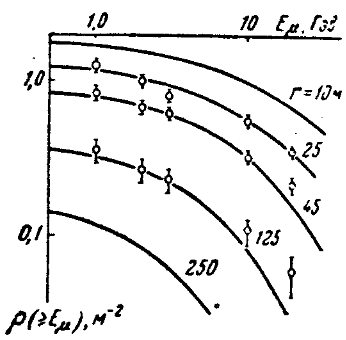

Fig. 4. - Energy spectra of muon fluxes at various distances from the air-shower core.

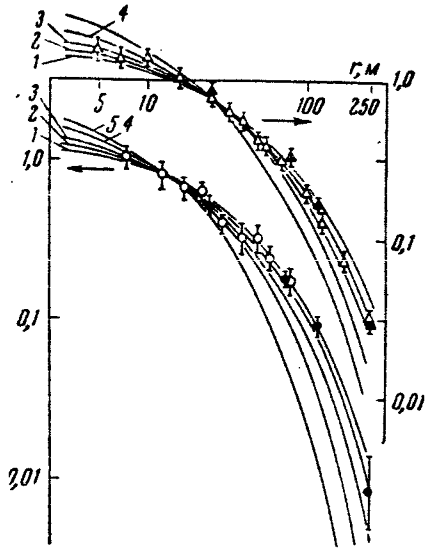

Fig. 3. - Spatial Distribution Functions of muon fluxes with $E_\mu = 5$ hev and $E_\mu = 10$ hev, corresponding to the functions F (x) of Fig. 2. - The left-hand ordinate axis is $\rho(r, E_\mu = 10\,\text{hev})\,m^{-2}$. the right-hand one $\rho(r, E_\mu = 5\,\text{hev})$, m^{-2}. The experimental data were borrowed from [10] :

Δ — [2]; $\circ$ — [3], $\bullet$, $\blacktriangle$ — [1].

The generation spectrum of pions, found from data on the spatial distribution of muons with energy of 5 and 10 hev, could be profitably utilized to obtain other data on the muon component of BAS, and compare them with the experiment. The computed spatial distribution of muons with $E_\mu = 0.5$ hev and $E_\mu = 40$ hev is plotted in Fig. 1 alongside with the experimental data of [7 — 9]. In Fig. 4 we represented the computed energy spectra of muons at various distances from the core of the shower alongside with the experimental data of [11].

Let us now make some remarks. The most reliably obtained course of the spatial distribution of muons is in the works [7 — 8], but the absolute value of density of muon fluxes may be subject to doubt on account of the difficulties in identifying muons in ground detectors. That is why the data from [7 — 8], brought out in Fig. 1, are normalized to the calculation at the point $r = 250$ m. In reference [11] the more reliable information concerns the the energy spectrum of muons (directly measured in the experiment, rather than the data on the spatial distribution of muons which differs somewhat from that obtained in [1 — 3]). Therefore, the comparison of the calculation is made with the energy spectrum of muons, while the experimental spectra at the distance r are normalized to calculations at the point $E_\mu = 2$ hev. Taking the above inaccuracies of the experiment into account, we may deem the agreement of calculation with the experiment satisfactory.

It is interesting to compare the number of pions obtained from the analysis of data on the muon component with the experimentally observed number of nuclearly-active particles in showers with the same number N of particles. Normalizing the computed curves for $\rho(r, E_\mu)$ and the experimental distributions of muon flux density, we may compute the factor b, and by the same token find the absolute number of pions generated in BAS with a given number N of particles. The number of pions, observed in a shower as nuclearly-active particles, is linked with the spectrum of pion generation as follows :

$$N_{\pi\,я-а}(E)\,dE = \int_0^\infty dH \cdot bE^{-\gamma-1}F(H)\exp(-H/L_{эфф})\,dE, \qquad (2)\ *$$

$$1/L_{эфф} = 1/L_{в_\circ} + 1/L_{p}.$$

The number of nuclearly-active pions in the energy range $5 \div 10$ hev, computed according to formula (2), was found to be 12 — 25 percent of the total number of nuclearly-active particles of same energies (taken from [6]).

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* [The Russian denotations should be read as follows : $N_{\pi\,я-а}$ = number of nucleO-active particles in ref. to π -mesons (pions); $L_{эфф} = L_{eff}$.]

which is in good agreement with the experiments on the number of charged and neutral nuclearly-active particles. According to (2), the share of pions in the number of nuclearly-active particles rises with the increase of energy, and at $10^{11} - 10^{12}$ ev it reaches the values $\geqslant 0.5$.

Therefore, the computation just conducted shows that it is possible to find a form of generation function of charged pions, which would give a good agreement with the available data on the spatial distribution of muons, on the their energy spectrum at various distances from the core of the shower, on the number of nuclearly-active pions in BAS and on the experimental distribution of transverse pion pulses in generation events.

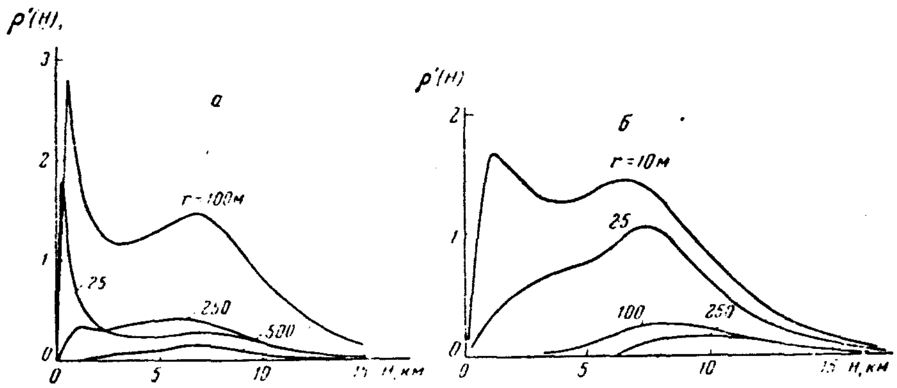

Fig. 5. - The muon birth function in height: a — for muons with $E_\mu = 0.5$ hev (at $r = 25$ m a density $0.1\rho(25)$ is plotted; δ — for muons with $E = 10$ hev (at $r = 250$ m the density $5\rho(250)$) is plotted; $\rho'(H)$ is the density of muons, observed at distances r from the axis of the shower, generated between H and H + dH.

The birth of muon flux as a function of height is plotted in Fig. 5, for a flux observed at a given distance from the axis of shower. For muons with energy $E \geqslant E_\mu$, it is possible to find the distances r_k from the axis of the shower, over which the main muon flux is collected from great heights. For $E_\mu = 0.5$ hev, the distance $r_k \approx 250$ m, for $E_\mu = 5$ hev, it is equal to $r_k \approx 40$ m and for $E_\mu = 10$ hev it constitutes $r_k \approx 25$ m. As may be seen, the distances r_k are significantly less than the mean radii of muon showers with a given E_μ. This means, that the density of muon fluxes must be proportional to the total number of muons in the shower, not only in the region $r \sim r_\mu$, radius of the muon shower, but in the entire region $r > r_k$.*

Another peculiarity of the generation function of muons resides in the comparatively weak absorption of the ion component (Λ = 200 ÷ 250 g/cm^2), which is prerequisite for the explanation of the observed experimental amount of low energy muons at distances $r > r_k$.

2.- POSSIBLE FLUCTUATIONS OF THE SPATIAL DISTRIBUTION FUNCTION OF MUONS

When observing individual broad air showers, deflections of the spatial distribution function of muons from average are possible; they are linked with the fluctuations in the development of a pion avalanche in the atmosphere. Certain predictions relative to fluctuations of spatial distribution function of muons can be made starting from the simplest model of pion avalanche development, when the form of the pion generation function does not vary and when the only thing that fluctuates is the depth of avalanche onset.**

The family of pion generation functions, plotted in Fig. 2, offers in this connection an example of pion avalanches formed at various atmosphere depths. To each of the function $F(x)$ of this family one may compare a depth x_0 of avalanche onset. We shall estimate, that x_0 corresponds to the depth of the first event of nuclear interaction of the primary particle. Then the probability of materializing the given pion generation function $F_{x_0}(x)$ (and by the same token of the corresponding spatial distribution of muons) is easy to connect with the interaction path λ of the primary particle with a fixed energy E_0:

$$W(x_0)\,dx_0 = \exp\left(-x_0/\lambda\right) dx_0/\lambda. \qquad (3)$$

Knowing the probability (3) and the dependence of muon density $\rho(r, E_\mu)$ on x_0, we may pass to the probability of muon density appearance in the shower with the given primary energy E_0. The computation of the

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* In the experiments of [3, 12] for measurement of muon flux fluctuations measurements were conducted at distances $r > r_h$, which nullifies the Nikol'skiy remark made in the review [6]. ** see note next page.

The calculation of this last probability was conducted by us for the family of curves $\rho(r, E_\mu = 10\,\mathrm{hev})$ of the type shown in Fig. 3. The value of the integral

$$\int_0^\infty F(x)\,dx$$

was taken as the first measure of primary energy. The result of the calculation is shown in Fig. 6 a.

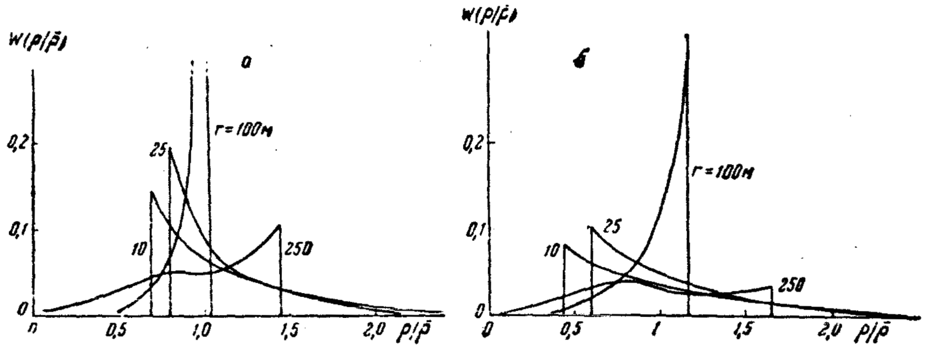

Fig. 6. - Fluctuations of the density of muon flux at given distance from shower axis, only linked with the fluctuations of the spatial distribution function: a — for showers with given primary energy; б — for showers with given number of particles.- The development of the showers is according to model adopted in ref. [5].

In experiments for measuring the fluctuations of muon fluxes on BAS, the total number N of particles is usually fixed. Generally speaking, this requirement may lead to redistribution of the depths x_0 of pion avalanches' onset. For example, in case of "hard" link between all the components of the shower, considered in reference [5], the assortment of showers with a given number N of particles, leads to the following distribution by x_0:

$$W(x_0) \sim \exp(-x_0/\lambda + \gamma x_0/\Lambda), \qquad (4)$$

where Λ is the path for particle absorption in the shower. In case of total independence in the development of the electron-photon component and of the muon component for a fixed total number N of particles, the distribution (3) is preserved.

** (from page 7).. When considering the development of a pion avalanche, such a model was better justified than when considering the development of the electron-photon component of BAS, when a great role may be played by fluctuations in energy transfer of the latter in separate nuclear interactior events.

In Fig. 6 $б$ we plotted the computed distribution of the muon density ρ ratio in an individual shower to the mean density $\bar{\rho}$ at the given distance from the axis of the shower ($\lambda = 80$ g/cm^2, $\gamma = 1.5$, $\Lambda = 200$ g/cm^2) according to (4). In order to separate the fluctuations, linked with the fluctuations of the spatial distribution function of muons, the showers with the given number of particles were normalized in the same fashion as was done in Fig. 6 a, that is by the value of the integral

$$\int_0^\infty F(x)\,dx$$

As may be seen, in case of "hard" connection between the development of the muon and the electron-photon components of the shower, the fluctuations of the spatial distribution of muons are more substantial. It is interesting to note, that the direction of the fluctuations of spatial distribution and of the total number of muons is in this case such, that at small distances from shower axis ,($r < 25$ m), the fluctuations of muon density $\rho/\bar{\rho}$ by comparison with the fluctuations of the total number of muons decrease, while at great distances,($r > 100$ m), they increase.

At present there are no sufficient experimental foundations for the selection of a specific variant of connection between the electron-photon and the muon components of a shower. One may only assert, on the basis of the observed width of fluctuations of muon flux density at a given distance from the axis of the shower (see [3]) and of comparison with the distribution of Fig. 6, that in the observed fluctuations the main part is played by the fluctuations of the total number of muons in a shower.

In conclusion, the author expresses his deep gratitude to Prof. S. N. Vernov and G. V. Khristiansen for discussing the results.

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**** THE END ****

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